Math 55 Discussion problems 25 Apr

1. The complete m-partite graph $K_{n_1,n_2,\ldots,n_m}$ has vertices partitioned into m subsets of $n_1, n_2, \ldots, n_m$ elements each, and vertices are adjacent if and only if they are in different subsets in the partition. Draw the following graphs.

   (a) $K_{1,2,3}$ (b) $K_{2,2,2}$ (c) $K_{1,2,2,3}$

2. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

3. Prove or disprove that there are always two vertices of the same degree in a finite multigraph having at least two vertices.

4. Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq \frac{v^2}{4}$.

5. Suppose that $2n$ tennis players compete in a round-robin tournament. Every player has exactly one match with every other player during $2n-1$ consecutive days. Every match has a winner and a loser. Use Hall's theorem to show that it is possible to select a winning player each day without selecting the same player twice.

6. The converse of a directed graph $G = (V, E)$, denoted by G^{conv}, is the directed graph (V, F), where the set F of edges of G^{conv} is obtained by reversing the direction of each edge in E. Show that the graph G is its own converse if and only if the relation associated with G is symmetric.